Gene Selection Using GeneSelectMMD

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1 Introduction

This document demonstrates how to use the *GeneSelectMMD* package to detect significant genes and to estimate false discovery rate (FDR), false non-discovery rate (FNDR), false positive rate (FPR), and false negative rate (FNR), for a real microarray data set based on the method proposed by Qiu et al. (2008). It also illustrates how to visualize the fit of the model proposed in Qiu et al. (2008) to the real microarray data set when the marginal correlations among subjects are zero or close to zero.

The *GeneSelectMMD* package is suitable for the case where there are only two tissue types and for the case where tissue samples within a tissue type are conditionally independent given the gene (c.f. Qiu et al., 2008).

2 Methods

MMD assumes that the marginal distribution of a gene profile is a mixture of 3-component multivariate normal distributions with special structure for mean vectors and covariance matrices. The 3 component distributions correspond to 3 gene clusters: (1) cluster of genes over-expressed in the treatment group; (2) cluster of genes non-differentially expressed; and (3) cluster of genes under-expressed in the treatment group. Specifically, the marginal density of a gene profile is assumed to be

$$f(\boldsymbol{x}|\boldsymbol{\theta}_{1},\boldsymbol{\theta}_{2},\boldsymbol{\theta}_{3}) = \pi_{1}f_{1}(\boldsymbol{x}|\boldsymbol{\theta}_{1}) + \pi_{2}f_{2}(\boldsymbol{x}|\boldsymbol{\theta}_{2}) + \pi_{3}f_{3}(\boldsymbol{x}|\boldsymbol{\theta}_{3}),$$

$$\pi_{1} + \pi_{2} + \pi_{3} = 1, \pi_{i} > 0, i = 1, 2, 3,$$

(1)

where π_1 , π_2 , π_3 are mixture proportions. The $m \times 1$ vector \boldsymbol{x} is a realization of the random vector \boldsymbol{X} that represents the transformed gene profile for a randomly selected gene over m tissue samples ($m = m_c + m_n$, where m_c is the number of abnormal tissue samples and m_n normal tissue samples); $\boldsymbol{\theta}_k$, is the parameter set for the k-th component distribution f_k , k = 1, 2, 3; and f_1 , f_2 , and f_3 are the density functions for multivariate Normal distributions with the mean vectors

$$\boldsymbol{\mu}_1 = \begin{pmatrix} \mu_{c1} \mathbf{1}_{m_c} \\ \mu_{n1} \mathbf{1}_{m_n} \end{pmatrix}, \quad \boldsymbol{\mu}_2 = \mu_2 \mathbf{1}_m, \quad \boldsymbol{\mu}_3 = \begin{pmatrix} \mu_{c3} \mathbf{1}_{m_c} \\ \mu_{n3} \mathbf{1}_{m_n} \end{pmatrix}.$$
(2)

and covariance matrices

$$\boldsymbol{\Sigma}_{1} = \begin{pmatrix} \sigma_{c_{1}}^{2} \boldsymbol{R}_{c_{1}} & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_{n_{1}}^{2} \boldsymbol{R}_{n_{1}} \end{pmatrix}, \quad \boldsymbol{\Sigma}_{2} = \sigma_{2}^{2} \boldsymbol{R}_{2}, \quad \boldsymbol{\Sigma}_{3} = \begin{pmatrix} \sigma_{c_{3}}^{2} \boldsymbol{R}_{c_{3}} & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_{n_{3}}^{2} \boldsymbol{R}_{n_{3}} \end{pmatrix}, \quad (3)$$

respectively, where correlation matrix

$$\boldsymbol{R}_{t} = (1 - \rho_{t}) \left[\boldsymbol{I}_{n_{t}} + \frac{\rho_{t}}{(1 - \rho_{t})} \boldsymbol{1}_{n_{t}} \boldsymbol{1}_{n_{t}}^{T} \right],$$
(4)

 $t = c_1, n_1, 2, c_3$, or n_3 . $n_t = m_c$ if $t = c_1$ or c_3 ; $n_t = m$ if t = 2; $n_t = m_n$ if $t = n_1$, or n_3 . Here we assume, without loss of generality, that the first m_c elements of the random vector **X** are for the abnormal tissue samples and the remaining m_n elements are for the normal tissue samples. Let $\boldsymbol{\theta}_1 = (\mu_{c_1}, \sigma_{c_1}^2, \rho_{c_1}, \mu_{n_1}, \sigma_{n_1}^2, \rho_{n_1})^T$, $\boldsymbol{\theta}_2 = (\mu_2, \sigma_2^2, \rho_2)^T, \boldsymbol{\theta}_3 = (\mu_{c_3}, \sigma_{c_3}^2, \rho_{c_3}, \mu_{n_3}, \sigma_{n_3}^2, \rho_{n_3})^T$. Note that $\mu_{c_1} > \mu_{n_1}$ for component 1 in which genes are overexpressed in abnormal tissue samples, and $\mu_{c_3} < \mu_{n_3}$ for component 3 where genes are underexpressed in abnormal samples. Our prior belief is that the majority of genes are usually non-differentially expressed, so we assume $\pi_2 > \pi_1$ and $\pi_2 > \pi_3$.

The model parameters can be estimated using the EM algorithm (Dempster et al., 1977).

The cluster membership of a gene is determined by the posterior probability that the gene belongs to a cluster given its gene profile. The posterior probability is a function of the 3 component marginal density functions and the mixing proportions:

$$Pr(\text{gene } i \in \text{cluster } k | \boldsymbol{x}_i) = \frac{\pi_k f_k(\boldsymbol{x}_i | \boldsymbol{\theta}_k)}{\pi_1 f_1(\boldsymbol{x}_i | \boldsymbol{\theta}_1) + \pi_2 f_2(\boldsymbol{x}_i | \boldsymbol{\theta}_2) + \pi_3 f_3(\boldsymbol{x}_i | \boldsymbol{\theta}_3)},$$

$$k = 1, 2, 3,$$
(5)

Specifically, a gene is assigned to the a gene cluster if the posterior probability that the gene belongs to that gene cluster given its gene profile is larger than the posterior probability that the gene belongs to other gene clusters given its gene profile:

$$k_0 = \arg \max_{k=1,2,3} \Pr(\text{gene } i \in C_k | \boldsymbol{x}_i).$$
(6)

An important task is to assess the performance of a gene selection method so that different methods can be objectively compared. To evaluate the performance of a gene selection method, investigators usually compare the error rates, such as FDR, FNDR, FPR, and FNR, via simulation studies. However, when analyzing a real microarray data set, investigators are more interested in what the values of FDR, FNDR, FPR, and FNR are for this specific real microarray set. It is challenging to estimate FDR, FNDR, FPR, and FNR for real microarray data sets since the true gene cluster membership is unknown for real data sets. However, model-based gene clustering methods, such as Bayesian hierarchical models and MMD, can provide such estimates since these error rates can be expressed as functions of marginal density functions and mixing proportions, where the model parameters and mixing proportions can be estimated from the real microarray data. It is easy to use MMD to estimate the four error rates since MMD describes the distributions of gene expression levels directly via the marginal distributions, while it is usually difficult to derive the marginal density functions for Bayesian hierarchical models.

It is of practical importance to evaluate if a model fits a real microarray data set well. If a model does not fit well for the real microarray data set, then it makes no sense to estimate the error rates based on the model. Although it is quite challenging to asses the goodness of fit for multivariate data, especially for non-normal multivariate data, it is possible to do so for some special cases. For example, when tissue samples are marginally independent, we could pool the gene expression levels across tissue samples for each type of tissue samples since they are all independent. We then could impose the theoretical density curve on the histogram of the pooled expression levels for each type of tissue samples. The parameters ρ_{c1} , ρ_{n1} , ρ_2 , ρ_{c3} , and ρ_{n3} indicates the marginal correlations among tissue samples. Assuming that the marginal correlations are ignorable, we then could produce such a plot to evaluate the goodness of fit of MMD to the real microarray data set.

3 Gene selection via *GeneSelectMMD*

The GeneSelectMMD includes four functions for gene selection: gsMMD.gsMMD.default, gsMMD2, and gsMMD2.default.

The functions gsMMD and gsMMD2 accept the object derived from the class of *Bioconductor*'s ExpressionSet as data input, while the functions gsMMD.default and gsMMD2.default accept data matrix as data input.

The functions gsMMD and gsMMD.default will provide initial 3-cluster gene partitions (cluster of genes over-expressed in treatment group, cluster of genes non-differentially expressed, and cluster of genes underexpressed in treatment group) based on the gene-wise two-sample t-test or two-sample Wilcoxon rank-sum test. In situations where the user would like to provide the initial 3-cluster partition other than that provided by the gene-wise two-sample t-test or two sample Wilcoxon rank-sum test, the functions gsMMD2 and gsMMD2.default could be used.

The rows of the input data matrix (or the data matrix derived from the object derived from the class ExpressionSet) are genes, while the columns are tissue samples. The tissue type of a tissue sample is indicated by the argument memSubjects, which is a $m \times 1$ vector, $m = m_c + m_n$, m_c is the number of tissue samples in the treatment group, and m_n is the number of tissue samples in the control group. memSubjects[j]=1 indicates the *j*-th tissue sample is from the treatment group. memSubjects[j]=0 indicates the *j*-th tissue sample is from the control group.

The output of gene selection functions include dat, memGenes, memGenes2, and para.

- dat is a data matrix with the same dimensions as the input data matrix. If no data transformation is performed, dat is the same as the input data matrix. Otherwise, it will be the transformed input data matrix.
- memGenes is a $G \times 1$ vector indicating the gene cluster membership, where G is the number of genes (i.e., the number of rows of the input data matrix). memGenes[g]=1 indicates the g-th gene is assigned to the cluster of genes over-expressed in treatment group; memGenes[g]=2 indicates the g-th gene is assigned to the cluster of genes non-differentially-expressed; memGenes[g]=3 indicates the g-th gene is assigned to the cluster of genes under-expressed in treatment group.
- memGenes2 is a variant of memGenes. memGenes2[g]=1 means the g-th gene is differentially expressed, while memGenes2[g]=0 means the g-th gene is non-differentially expressed, $g = 1, \ldots, G$.
- para is a 18×1 vector of parameters (π₁, π₂, π₃, μ_{c1}, σ²_{c1}, ρ_{c1}, μ_{n1}, σ²_{n1}, ρ_{n1}, μ₂, σ²₂, ρ₂, μ_{c3}, σ²_{c3}, ρ_{c3}, μ_{n3}, σ²_{n3}, ρ_{n3},) for the model described in Equations (1)- (4), which can be estimated by the EM algorithm. para[1], para[2], and para[3] are the cluster proportions for the 3 gene clusters (over-expressed, non-differentially expressed, and under-expressed). para[4], para[5], and para[6] are the marginal mean, variance, and correlation of gene expression levels of cluster 1 (up-regulated genes) for diseased subjects; para[7], para[8], and para[9] are the marginal mean, variance, and correlation of gene expression levels of cluster 3 (up-regulated genes) for cluster 2 (non-differentially expressed genes); para[13], para[14], and para[15] are the marginal mean, variance, and correlation of gene expression levels of cluster 3 (up-regulated genes) for diseased subjects; para[18] are the marginal mean, variance, and correlation of gene expression levels of cluster 3 (up-regulated genes) for diseased subjects; para[18] are the marginal mean, variance, and correlation of gene expression levels of cluster 3 (up-regulated genes) for diseased subjects; para[16], para[17], and para[18] are the marginal mean, variance, and correlation of gene expression levels of cluster 3 (up-regulated genes) for diseased subjects; para[16], para[17], and para[18] are the marginal mean, variance, and correlation of gene expression levels of cluster 3 (up-regulated genes) for diseased subjects; para[16], para[17], and para[18] are the marginal mean, variance, and correlation of gene expression levels of cluster 3 (up-regulated genes) for diseased subjects; para[16], para[17], and para[18] are the marginal mean, variance, and correlation of gene expression levels of cluster 3 (up-regulated genes) for non-diseased subjects.

Note that genes in cluster 2 are non-differentially expressed across abnormal and normal tissue samples. Hence there are only 3 parameters for cluster 2.

For example, to obtain differentially expressed genes for the ALL data (Chiaretti et al., 2004), we can run either

```
> library(GeneSelectMMD)
> library(ALL)
> data(ALL)
> eSet1 <- ALL[1:100, ALL$BT == "B3" | ALL$BT == "T2"]
> mem.str <- as.character(eSet1$BT)</pre>
```

```
> nSubjects <- length(mem.str)</pre>
> memSubjects <- rep(0, nSubjects)</pre>
> memSubjects[mem.str == "T2"] <- 1</pre>
> obj.gsMMD <- gsMMD(eSet1, memSubjects, transformFlag = TRUE,</pre>
      transformMethod = "boxcox", scaleFlag = TRUE, quiet = FALSE)
Programming is running. Please be patient...
Data transformation ( boxcox ) performed
Gene profiles are scaled so that they have mean zero and variance one!
Programming is running. Please be patient...
******* initial parameter estimates method>> Ttest ******
paraIniMat[,i]>>
    pi.1
                       pi.3
                                mu.c1 sigma2.c1
                                                  rho.c1
                                                             mu.n1 sigma2.n1
              pi.2
    0.050
             0.870
                       0.080
                                0.556
                                          0.604
                                                    0.003
                                                            -0.363
                                                                       0.947
   rho.n1
             mu.2 sigma2.2
                               rho.2
                                          mu.c3 sigma2.c3
                                                            rho.c3
                                                                       mu.n3
   0.001
             0.000
                      1.000
                               0.000 -0.659
                                                   0.513
                                                             0.097
                                                                       0.430
sigma2.n3
            rho.n3
   0.812
             0.028
llkhVec>>
   Ttest
-5284.364
Initial parameter estimates>>
          Ttest
          0.050
pi.1
pi.2
          0.870
pi.3
          0.080
mu.c1
          0.556
sigma2.c1 0.604
rho.c1
          0.003
mu.n1
         -0.363
sigma2.n1 0.947
rho.n1
          0.001
mu.2
          0.000
sigma2.2 1.000
rho.2
          0.000
mu.c3
         -0.659
sigma2.c3 0.513
rho.c3
          0.097
mu.n3
          0.430
sigma2.n3 0.812
rho.n3
          0.028
Initial loglikelihood>>
    Ttest
-5308.485
Final parameter estimates based on initial estimates>>
          Ttest
```

```
4
```

<pre>pi.1 pi.2 pi.3 mu.c1 sigma2.c1 rho.c1 mu.n1 sigma2.n1 rho.n1 mu.2 sigma2.2 rho.2 mu.c3 sigma2.c3 rho.c3 mu.n3 sigma2.n3 rho.n3</pre>	0.049 0.906 0.044 0.557 0.563 -0.069 -0.363 0.908 -0.045 0.000 0.974 -0.027 -0.808 0.292 0.061 0.527 0.715 -0.023									
Final log Ttest -5284.364	likelihood	based on in	itial est	timates>>						
Final nam	motor ogti	mator>>								
rinai para	ameter esti	mates//	mu o1	airma 2 al	who of		airma0 m1			
0 0/0	0 906	0.044		51gma2.C1	-0.069	_0 363	0 008			
rho n1	0.900 mil 2	0.044 sigma2 2	rho 2	mil c3	0.009 sigma2_c3	rho c3	mii n3			
-0 045	0 000	0 974	-0 027	-0 808	0 292	0 061	0 527			
0.040 sigma2 n3	rho n3	0.974	0.021	0.000	0.292	0.001	0.521			
0.715	-0.023									
Final log Ttest -5284.364	likelihood>	·>								
*******	*******	<**********	*******	********	*****					
> para <- obj.gsMMD\$para > print(round(para, 3))										
pi.1	pi.2	pi.3	mu.c1	sigma2.c1	rho.c1	mu.n1	sigma2.n1			
0.049	0.906	0.044	0.557	0.563	-0.069	-0.363	0.908			
rho.n1	mu.2	sigma2.2	rho.2	mu.c3	sigma2.c3	rho.c3	mu.n3			
-0.045	0.000	0.974	-0.027	-0.808	0.292	0.061	0.527			
sigma2.n3	rho.n3									
0.715	-0.023									
or										
> library(GeneSelectMMD)										
> library(ALL)										
> data(ALL)										
> eSet1 <- ALL[1:100, ALL\$BT == "B3" ALL\$BT == "T2"]										

```
> mat <- exprs(eSet1)</pre>
> mem.str <- as.character(eSet1$BT)</pre>
> nSubjects <- length(mem.str)</pre>
> memSubjects <- rep(0, nSubjects)</pre>
> memSubjects[mem.str == "T2"] <- 1</pre>
> obj.gsMMD <- gsMMD.default(mat, memSubjects, iniGeneMethod = "Ttest",</pre>
      transformFlag = TRUE, transformMethod = "boxcox", scaleFlag = TRUE)
Programming is running. Please be patient...
Programming is running. Please be patient...
> para <- obj.gsMMD$para</pre>
> print(round(para, 3))
                                  mu.c1 sigma2.c1
    pi.1
              pi.2
                        pi.3
                                                     rho.c1
                                                                mu.n1 sigma2.n1
                                  0.557 0.563
                                                     -0.069
   0.049
              0.906
                        0.044
                                                               -0.363
                                                                           0.908
                                  rho.2
                                                             rho.c3
   rho.n1
              mu.2 sigma2.2
                                            mu.c3 sigma2.c3
                                                                           mu.n3
   -0.045
             0.000
                     0.974
                                 -0.027 -0.808 0.292
                                                             0.061
                                                                           0.527
sigma2.n3
            rho.n3
            -0.023
   0.715
```

If we would like to provide the initial 3-cluster partition via the two sample Wilcoxon rank-sum test, then we can run either

```
> library(GeneSelectMMD)
> library(ALL)
> data(ALL)
> eSet1 <- ALL[1:100, ALL$BT == "B3" | ALL$BT == "T2"]
> mem.str <- as.character(eSet1$BT)</pre>
> nSubjects <- length(mem.str)</pre>
> memSubjects <- rep(0, nSubjects)</pre>
> memSubjects[mem.str == "T2"] <- 1</pre>
> myWilcox <- function(x, memSubjects, alpha = 0.05) {</pre>
      xc <- x[memSubjects == 1]</pre>
+
      xn <- x[memSubjects == 0]</pre>
+
      m <- sum(memSubjects == 1)</pre>
+
      res <- wilcox.test(x = xc, y = xn, conf.level = 1 - alpha)</pre>
+
+
      res2 <- c(res$p.value, res$statistic - m * (m + 1)/2)
+
      names(res2) <- c("p.value", "statistic")</pre>
      return(res2)
+
+ }
> mat <- exprs(eSet1)</pre>
> tmp <- t(apply(mat, 1, myWilcox, memSubjects = memSubjects))</pre>
> colnames(tmp) <- c("p.value", "statistic")</pre>
> memIni <- rep(2, nrow(mat))</pre>
> memIni[tmp[, 1] < 0.05 & tmp[, 2] > 0] <- 1
> memIni[tmp[, 1] < 0.05 & tmp[, 2] < 0] <- 3
> print(table(memIni))
memIni
1 2 3
7 85 8
```

> obj.gsMMD <- gsMMD2(eSet1, memSubjects, memIni, transformFlag = TRUE,</pre> transformMethod = "boxcox", scaleFlag = TRUE, quiet = FALSE) + Programming is running. Please be patient... Data transformation (boxcox) performed Gene profiles are scaled so that they have mean zero and variance one! Programming is running. Please be patient... Initial parameter estimates>> pi.1 pi.2 pi.3 mu.c1 sigma2.c1 rho.c1 mu.n1 sigma2.n1 0.070 0.850 0.080 0.494 0.734 0.016 -0.322 0.927 mu.2 sigma2.2 rho.n1 rho.2 mu.c3 sigma2.c3 rho.c3 mu.n3 0.006 0.000 1.000 0.000 -0.654 0.546 0.096 0.426 sigma2.n3 rho.n3 0.795 0.030 Initial loglikelihood>> [1] -5313.98 Final parameter estimates>> pi.2 pi.1 pi.3 mu.c1 sigma2.c1 rho.c1 mu.n1 sigma2.n1 0.049 0.906 0.044 0.557 0.563 -0.069 -0.363 0.908 rho.n1 mu.2 sigma2.2 rho.2 mu.c3 sigma2.c3 rho.c3 mu.n3 -0.027 -0.808 0.292 -0.045 0.000 0.974 0.061 0.527 sigma2.n3 rho.n3 -0.023 0.715 Final loglikelihood>> [1] -5284.364 > para <- obj.gsMMD\$para</pre> > print(round(para, 3)) mu.c1 sigma2.c1 mu.n1 sigma2.n1 pi.1 pi.2 pi.3 rho.c1 0.049 0.906 0.044 0.557 0.563 -0.069 -0.363 0.908 rho.n1 mu.2 sigma2.2 rho.2 mu.c3 sigma2.c3 rho.c3 mu.n3 -0.027 -0.045 0.000 0.974 -0.808 0.292 0.061 0.527 rho.n3 sigma2.n3 0.715 -0.023 or > library(GeneSelectMMD) > library(ALL) > data(ALL) > eSet1 <- ALL[1:100, ALL\$BT == "B3" | ALL\$BT == "T2"] > mat <- exprs(eSet1)</pre> > mem.str <- as.character(eSet1\$BT)</pre> > nSubjects <- length(mem.str)</pre> > memSubjects <- rep(0, nSubjects)</pre>

```
> memSubjects[mem.str == "T2"] <- 1</pre>
> myWilcox <- function(x, memSubjects, alpha = 0.05) {</pre>
+
      xc <- x[memSubjects == 1]</pre>
      xn <- x[memSubjects == 0]</pre>
+
+
      m <- sum(memSubjects == 1)</pre>
      res <- wilcox.test(x = xc, y = xn, conf.level = 1 - alpha)</pre>
+
      res2 <- c(res$p.value, res$statistic - m * (m + 1)/2)
      names(res2) <- c("p.value", "statistic")</pre>
+
+
      return(res2)
+ }
> tmp <- t(apply(mat, 1, myWilcox, memSubjects = memSubjects))</pre>
> colnames(tmp) <- c("p.value", "statistic")</pre>
> memIni <- rep(2, nrow(mat))</pre>
> memIni[tmp[, 1] < 0.05 & tmp[, 2] > 0] <- 1
> memIni[tmp[, 1] < 0.05 & tmp[, 2] < 0] <- 3
> print(table(memIni))
memIni
1 2
       3
7 85
       8
> obj.gsMMD <- gsMMD2.default(mat, memSubjects, memIni = memIni,</pre>
      transformFlag = TRUE, transformMethod = "boxcox", scaleFlag = TRUE)
Programming is running. Please be patient...
Programming is running. Please be patient...
> para <- obj.gsMMD$para</pre>
> print(round(para, 3))
     pi.1
                                     mu.c1 sigma2.c1
                pi.2
                           pi.3
                                                          rho.c1
                                                                     mu.n1 sigma2.n1
    0.049
                          0.044
                                                          -0.069
                                                                     -0.363
               0.906
                                     0.557
                                                0.563
                                                                                 0.908
   rho.n1
                mu.2
                       sigma2.2
                                     rho.2
                                                mu.c3 sigma2.c3
                                                                     rho.c3
                                                                                 mu.n3
   -0.045
               0.000
                          0.974
                                    -0.027
                                               -0.808
                                                           0.292
                                                                      0.061
                                                                                 0.527
sigma2.n3
              rho.n3
    0.715
              -0.023
```

Actually, the two sample Wilcoxon rank-sum test is implemented in the *GeneSelectMMD*. The above code is used as an illustration on how to use the functions gsMMD2 and gsMMD2.default.

Note that the speed of the four functions is slow for large data sets. So we recommend that the program be in background mode if it is run in Unix or Linux environment.

4 Error rates estimated for real microarray data sets

For real microarray data sets, the classical gene selection methods, such as two-sample t-test, two-sample Wilcoxon rank-sum test, could not provide the estimates of error rates such as false discovery rate (denoted as FDR; it is the percentage of nondifferentially expressed genes among selected genes), false non-discovery rate (denoted as FNDR; it is the percentage of differentially expressed genes among unselected genes), false positive rate (denoted as FPR; it is the percentage of selected genes among nondifferentially expressed genes), and false negative rate (denoted as FNR; it is the percentage of un-selected genes among differentially expressed genes), since the true gene cluster membership is unknown.

However, model-based gene selection methods (e.g., eLNN and eGG proposed by Lo and Gottardo (2007), and the method proposed by Qiu et al.'s (2008)) could easily estimate these error rates, since these error rates are functions of some probabilities which are in turn the functions of model parameters.

The function errRates is used to estimate FDR, FNDR, FPR, and FNR based on the objects returned by the four gene selection functions mentioned in the previous section. This function returns a 4×1 vector with elements FDR, FNDR, FPR, and FNR.

For example,

rho.2

```
> library(GeneSelectMMD)
> library(ALL)
> data(ALL)
> eSet1 <- ALL[1:100, ALL$BT == "B3" | ALL$BT == "T2"]
> mem.str <- as.character(eSet1$BT)</pre>
> nSubjects <- length(mem.str)</pre>
> memSubjects <- rep(0, nSubjects)</pre>
> memSubjects[mem.str == "T2"] <- 1</pre>
> obj.gsMMD <- gsMMD(eSet1, memSubjects, transformFlag = TRUE,
      transformMethod = "boxcox", scaleFlag = TRUE, quiet = FALSE)
+
Programming is running. Please be patient...
Data transformation ( boxcox ) performed
Gene profiles are scaled so that they have mean zero and variance one!
Programming is running. Please be patient...
******* initial parameter estimates method>> Ttest
                                                     ******
paraIniMat[,i]>>
    pi.1
                         pi.3
              pi.2
                                  mu.c1 sigma2.c1
                                                     rho.c1
                                                                mu.n1 sigma2.n1
   0.050
              0.870
                        0.080
                                  0.556
                                            0.604
                                                      0.003
                                                               -0.363
                                                                          0.947
   rho.n1
              mu.2
                     sigma2.2
                                  rho.2
                                            mu.c3 sigma2.c3
                                                               rho.c3
                                                                          mu.n3
   0.001
                        1.000
                                  0.000
                                           -0.659
                                                                          0.430
              0.000
                                                      0.513
                                                                0.097
sigma2.n3
             rho.n3
   0.812
              0.028
llkhVec>>
   Ttest
-5284.364
Initial parameter estimates>>
           Ttest
pi.1
           0.050
           0.870
pi.2
pi.3
           0.080
mu.c1
           0.556
sigma2.c1
           0.604
           0.003
rho.c1
          -0.363
mu.n1
sigma2.n1
          0.947
rho.n1
           0.001
mu.2
           0.000
sigma2.2
           1.000
           0.000
```

mu.c3 -0.659 sigma2.c3 0.513 rho.c3 0.097 mu.n3 0.430 sigma2.n3 0.812 rho.n3 0.028 Initial loglikelihood>> Ttest -5308.485 Final parameter estimates based on initial estimates>> Ttest pi.1 0.049 pi.2 0.906 pi.3 0.044 0.557 mu.c1 sigma2.c1 0.563 rho.c1 -0.069 mu.n1 -0.363 sigma2.n1 0.908 rho.n1 -0.045 mu.2 0.000 sigma2.2 0.974 rho.2 -0.027 mu.c3 -0.808 sigma2.c3 0.292 rho.c3 0.061 mu.n3 0.527 sigma2.n3 0.715 rho.n3 -0.023 Final loglikelihood based on initial estimates>> Ttest -5284.364 Final parameter estimates>> pi.1 pi.2 pi.3 mu.c1 sigma2.c1 rho.c1 mu.n1 sigma2.n1 0.049 0.906 0.044 0.557 0.563 -0.069 -0.363 0.908 mu.n3 rho.n1 mu.2 sigma2.2 rho.2 mu.c3 sigma2.c3 rho.c3 -0.045 0.000 0.974 -0.027 -0.808 0.292 0.061 0.527 sigma2.n3 rho.n3 0.715 -0.023 Final loglikelihood>> Ttest -5284.364 ****** > print(round(errRates(obj.gsMMD), 3)) FDR FNDR FPR FNR

0.011 0.005 0.001 0.048

5 Visualizing the fit of the model to a real microarray data set

In general, it is difficult to visualize the fit of the model to a real microarray data set since the data are in high-dimensional space. However, it is possible to do so for the special case where the tissue samples within a tissue type are marginally independent. In this case, we can pool all gene expression levels together since they are all independent, and regard them as one-dimensional data. Next, we can impose the density estimate onto the histogram of this pooled data.

The function plotHistDensity is used for such purpose. The following R code illustrates the usage of plotHistDensity:

```
> library(GeneSelectMMD)
> library(ALL)
> data(ALL)
> eSet1 <- ALL[1:100, ALL$BT == "B3" | ALL$BT == "T2"]
> mem.str <- as.character(eSet1$BT)</pre>
> nSubjects <- length(mem.str)</pre>
> memSubjects <- rep(0, nSubjects)</pre>
> memSubjects[mem.str == "T2"] <- 1</pre>
> obj.gsMMD <- gsMMD(eSet1, memSubjects, transformFlag = TRUE,
      transformMethod = "boxcox", scaleFlag = TRUE, quiet = FALSE)
+
Programming is running. Please be patient...
Data transformation ( boxcox ) performed
Gene profiles are scaled so that they have mean zero and variance one!
Programming is running. Please be patient...
******* initial parameter estimates method>> Ttest
                                                     ******
paraIniMat[,i]>>
                                  mu.c1 sigma2.c1
    pi.1
              pi.2
                         pi.3
                                                     rho.c1
                                                                mu.n1 sigma2.n1
   0.050
              0.870
                        0.080
                                  0.556
                                            0.604
                                                      0.003
                                                               -0.363
                                                                          0.947
   rho.n1
              mu.2
                     sigma2.2
                                  rho.2
                                            mu.c3 sigma2.c3
                                                               rho.c3
                                                                          mu.n3
   0.001
              0.000
                        1.000
                                  0.000
                                           -0.659
                                                      0.513
                                                                0.097
                                                                          0.430
sigma2.n3
             rho.n3
   0.812
              0.028
llkhVec>>
   Ttest
-5284.364
Initial parameter estimates>>
           Ttest
pi.1
           0.050
pi.2
           0.870
pi.3
           0.080
           0.556
mu.c1
sigma2.c1
           0.604
rho.c1
           0.003
mu.n1
          -0.363
sigma2.n1 0.947
```

rho.n1 mu.2 sigma2.2 rho.2 mu.c3 sigma2.c3 rho.c3 mu.n3 sigma2.n3 rho.n3	0.001 0.000 1.000 0.000 -0.659 0.513 0.097 0.430 0.812 0.028						
Initial 10	oglikelihoo	d>>					
Ttest							
-5308.485							
Final para pi.1 pi.2 pi.3 mu.c1 sigma2.c1 rho.c1 mu.n1 sigma2.n1 rho.n1 mu.2 sigma2.2 rho.2 mu.c3 sigma2.c3 rho.c3 mu.n3 sigma2.n3 rho.n3	Ameter esti Ttest 0.049 0.906 0.044 0.557 0.563 -0.069 -0.363 0.908 -0.045 0.000 0.974 -0.027 -0.808 0.292 0.061 0.527 0.715 -0.023	mates based	on initi	ial estimat	ces>>		
110.110	0.020						
Final log	likelihood	based on in	itial est	cimates>>			
-5284.364							
Final para	ameter esti	.mates>>					
p1.1	p1.2	p1.3	mu.cl	sigma2.cl	rno.cl	mu.ni	sigma2.nl
0.049	0.900	0.044	0.557	0.503	-0.009	-0.303	0.908
-0.045	0 000	0 97/	-0 027	-0 808	0 202	0 061	0 527
sigma2 n3	rho n3	0.011	0.021	0.000	0.202	0.001	0.021
0.715	-0.023						
Final log Ttest -5284.364	likelihood>	~>					

```
> para <- obj.gsMMD$para</pre>
> print(round(para, 3))
                                mu.c1 sigma2.c1
                                                            mu.n1 sigma2.n1
    pi.1
              pi.2
                       pi.3
                                                  rho.c1
   0.049
             0.906
                      0.044
                                0.557
                                         0.563
                                                  -0.069
                                                           -0.363
                                                                      0.908
  rho.n1
              mu.2
                   sigma2.2
                                rho.2
                                         mu.c3 sigma2.c3
                                                           rho.c3
                                                                      mu.n3
   -0.045
             0.000
                      0.974
                               -0.027
                                        -0.808
                                                   0.292
                                                            0.061
                                                                      0.527
sigma2.n3
            rho.n3
            -0.023
   0.715
> print(round(errRates(obj.gsMMD), 3))
  FDR FNDR
             FPR
                  FNR
0.011 0.005 0.001 0.048
> plotHistDensity(obj.gsMMD, plotFlag = "case", mytitle = "Histogram of gene expression levels for T2\n.
     plotComponent = TRUE, x.legend = c(0.8, 3), y.legend = c(0.3, 3)
+
+
         0.4), numPoints = 500, cex.main = 1, cex = 1)
```



6 Discussion

The speeds of the four gene selection functions described in Section 3 are slow. One way to improve the speed is to embed Fortran or C code in the R code. We will take such an approach in the future version of the GeneSelectMMD.

References

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- [2] Dempster, A., Laird, N., and Rubin, D. Likelihood from incomplete data via the em algorithm. Journal of the Royal Statistical Society, Series B, 39(1):1–38, 1977.
- [3] Lo, K. and Gottardo, R. Flexible empirical bayes models for differential gene expression. *Bioinformatics*, 23:328–335, 2007.
- [4] Qiu, W.-L., He, W., Wang, X.-G., and Lazarus, R. A marginal mixture model for selecting differentially expressed genes across two types of tissue samples. *The International Journal of Biostatistics*, 4(1):Article 20, 2008.